Abelian projection and studies of gauge-variant quantities in the lattice QCD without gauge fixing

Sergei V. SHABANOV [1]

Institute for Theoretical Physics, Free University of Berlin, Arnimallee 14, WE 2, D-14195, Berlin, Germany

Abstract

We suggest a new (dynamical) Abelian projection of the lattice QCD. It contains no gauge condition imposed on gauge fields so that Gribov copying is avoided. Configurations of gauge fields that turn into monopoles in the Abelian projection can be classified in a gauge invariant way. In the continuum limit, the theory respects the Lorentz invariance. A similar dynamical reduction of the gauge symmetry is proposed for studies of gauge-variant correlators (like a gluon propagator) in the lattice QCD. Though the procedure is harder for numerical simulations, it is free of gauge-fixing artifacts, like the Gribov horizon and copies.

1. One of the important features of the QCD confinement is the existence of a stable chromoelectrical field tube connecting two color sources (quark and antiquark). Numerical studies of the gluon field energy density between two color sources leave no doubt that such a tube exists. However, a mechanism which could explain its stability is still unknown.

It is believed that some specific configurations (or excitations) of gauge fields are responsible for the QCD confinement, meaning that they give a main contributions to the QCD string tension. Numerical simulations of the lattice QCD shows that Abelian (commutative) configurations of gauge potentials completely determine the string tension in the full non-Abelian gauge theory [1]. This phenomenon is known as the Abelian dominance. Therefore one way of constructing effective dynamics of the configurations relevant to the QCD confinement is the Abelian projection [2] when the full non-Abelian gauge group SU(3) is restricted to its maximal Abelian subgroup (the Cartan subgroup) U(1)×U(1) by a gauge fixing. Though dynamics of the above gauge field configuration cannot be gauge dependent, a right choice of a guage condition may simplify its description.

There is a good reason, supported by numerical simulations [3], [4], to believe that the sought configurations turn into magnetic monopoles in the effective Abelian theory, and the confinement can be due to the dual mechanism [5]: The Coulomb field of electric charges is squized into a tube, provided monopole-antimonopole pair form a condensate like the Cooper pairs in superconductor.

It is important to realize that the existence of monopoles in the effective Abelian theory is essentially due to the gauge fixing, in fact, monopoles are singularities of the

[1] Alexander von Humboldt fellow;
on leave from *Laboratory of Theoretical Physics, JINR, Dubna, Russia*
present address: *Department of theoretical physics, University of Valencia, Dr. Moliner 50, Valencia (Burjassot), E-46100*

gauge fixing. Note that monopoles cannot exist as stable excitations in pure gauge theory with simply connected group like SU(3). Since the homotopy groups of SU(3) and of U(1)×U(1) are different (the one of SU(3) is trivial), a gauge condition restricting SU(3) to U(1)×U(1) should have singularities which can be identified as monopoles [2]. A dynamical question is to verify whether all configurations of non-Abelian gauge fields relevant to the confinement (in the aforementioned sense) are "mapped" on monopoles of the Abelian theory (the monopole domimance [4]). It appears that monopole dynamics may depend on the projection recipe [6]. There are indications that some Abelian projections exhibit topological singularities other than magnetic monopoles [7].

Though the lattice QCD is, up to now, the only relible tool for studying monopole dynamics, the true theory must be continuous and respect the Lorentz invariance. In this regard, Abelian projections based on Lotentz invariant gauge conditions play a distinguished role. For example, the gauge can be chosen as follows $D_\mu^H A_\mu^{off} = 0$ where $D_\mu^H = \partial_\mu + igA_\mu^H$, A_μ^H are Cartan (diagonal) components of guage potentials A_μ, while A_μ^{off} are its non-Cartan (off-diagonal) components. This gauge restricts the gauge symmetry to the maximal Abelian (Cartan) subrgoup and is manifestly Lorentz invariant. The lattice version of the corresponding Abelian theory is known as the maximal Abelian projection. The above homotopy arguments can be implemented to this gauge to show that it has topological singularities and Gribov's copying [9] (in the continuum theory, zero boundary conditions at infinity have to be imposed [10]). The Gribov copying makes additional difficulties for describing monopole dynamics (even in the lattice gluodynamics [11]).

In this letter, a new (dynamical) Abelian projection is proposed. It involves no gauge condition to be imposed on gauge fields. The effective Abelian theory appears to be nonlocal, though it can be made local at the price of having some additional (ghost) fields. All configurations of gauge fields that turn into magnetic monopoles in the effective Abelian theory are classified in a gauge invariant way. The effective Abelian theory fully respects the Lorentz symmetry and the Gribov problem is avoided.

Another important aspect of the QCD confinement is the absence of propagating color charges, meaning that a nonperturbative propagator of colored particles, gluons or quarks, has no usual poles in the momentum space. It has been argued that such a behavior of a gluon propagator in the Coulomb gauge could be due to an influence of the so called Gribov horizon on long-wave fluctuations of gauge fields [9], [12]. The result obviously depends on the gauge chosen, which makes it not very reliable.

The situation looks more controversial if one recalls that a similar qualitative behavior of the gluon propagator has been found in the study of Schwinger-Dyson equations [13]. In this approach, the Gribov ambiguities have not been accounted for. So, the specific pole structure of the gluon propagator occurred through a strong self-interaction of gauge fields.

In this letter, we would also like to propose a method for how to study gauge-variant quantities, like a gluon propagator, in the lattice QCD, avoiding any explicit gauge fixing. The method is, hence, free of all the aforementioned gauge fixing artifacts. It gives a hope that dynamical contributions (self-interaction of gauge fields) to the pole structure of the gluon propagator can be separated from the kinematical (gauge-fixing) ones.

2. To single out monopoles in non-Abelian gauge theory, one fixes partially a gauge so that the gauge-fixed theory possesses an Abelian gauge group being a maximal Abelian subgroup of the initial gauge group. The lattice formulation of the Abelian projection has been given in [8].

The idea is to choose a function $R(n)$ of link variables $U_\mu(n)$, n runs over lattice sites, such that

$$R(n) \to g(n)R(n)g^{-1}(n) \tag{1}$$

under gauge transformations of the link variables

$$U_\mu(n) \to g(n)U_\mu(n)g^{-1}(n+\hat{\mu}) \ , \tag{2}$$

where $g(n) \in G$, G is a compact gauge group, and $\hat{\mu}$ is a unit vector in the μ-direction. A gauge is chosen so that R becomes an element of the Cartan subalgebra H, a maximal Abelian subalgebra of a Lie algebra X of the group G. In a matrix representation, the gauge condition means that off-diagonal elements of R are set to be zero. Clearly, the gauge fixing is not complete. A maximal Abelian subgroup G_H of G remains as a gauge group because the adjoint action (1) of G_H leaves elements $R \in H$ untouched.

A configuration $U_\mu(n)$ contains monopoles if the corresponding matrix $R(n)$ has two coinciding eigenvalues. So, by construction, dynamics of monopoles appears to be gauge-dependent, or projection-dependent. It varies from gauge to gauge, from one choice of R to another [6]. Yet, the monopole singularities are not the only ones in some Abelian projections [7]. In addition, Abelian projections may suffer off the Gribov ambiguities [11].

To restrict the full gauge symmetry to its maximal Abelian part and, at the same time, to avoid imposing a gauge condition on link variables, we shall use a procedure similar to the one discussed in [14] in the framework of continuum field theory. A naive continuum limit of our procedure poses some difficulties. To resolve them, a corresponding operator formalism has to be developed. It has been done in [15] for a sufficiently large class of gauge theories.

Consider a complex Grassmann field $\psi(n)$ (a fermion ghost) that realizes the adjoint representation of the gauge group:

$$\psi(n) \quad \to \quad g(n)\psi(n)g^{-1}(n) \ , \tag{3}$$
$$\psi^*(n) \quad \to \quad g(n)\psi^*(n)g^{-1}(n) \ . \tag{4}$$

Let the fermion ghost be coupled to gauge fields according to the action

$$S_f = \sum_{n,\mu} tr D_\mu\psi^*(n)D_\mu\psi(n) \ , \tag{5}$$

where $D_\mu\psi(n) = \psi(n+\hat{\mu}) - U_\mu^{-1}(n)\psi(n)U_\mu(n)$ is the lattice covariant derivative in the adjoint representation. We assume that $\psi(n) = \psi_i(n)\lambda_i$, where λ_i is a matrix representation of a basis in X normalized as $tr\ \lambda_i\lambda_j = \delta_{ij}$, and $\psi_i(n)$ are complex Grassmann variables. The partition function of the fermion ghost field reads

$$Z_f(\beta) = \int \prod_n \left(d\psi^*(n)d\psi(n)\right) e^{-\beta S_f} = \det \beta D_\mu^T D_\mu \ , \tag{6}$$

where the integration over Grassmann variables is understood, and D_μ^T denotes a transposition of D_μ with respect to a scalar product induced by $\sum_{n,\mu} tr$ in (5). Note that the action (5) can be written in the form $S_f = \sum \psi^* D_\mu^T D_\mu \psi$.

Consider a pair of real Lie-algebra-valued scalar fields $\varphi(n)$ and $\phi(n)$ (boson ghosts) with an action

$$S_b = \frac{1}{2} \sum_{n,\mu} tr \left[(D_\mu \phi(n))^2 + (D_\mu \varphi(n))^2 \right] \ . \tag{7}$$

The action (7) is invariant under the gauge transformation (2), provided

$$\phi(n) \rightarrow g(n)\phi(n)g^{-1}(n) \ , \tag{8}$$

$$\varphi(n) \rightarrow g(n)\varphi(n)g^{-1}(n) \ . \tag{9}$$

The boson ghost partition function is

$$Z_b(\beta) = \int \prod_n \left(\frac{d\varphi(n) d\phi(n)}{(2\pi)^{\dim G}} \right) e^{-\beta S_b} = (\det \beta D_\mu^T D_\mu)^{-1} \ . \tag{10}$$

We have the identity

$$Z_b(\beta) Z_f(\beta) = 1 \ . \tag{11}$$

By making use of this identity, the partition function of gauge fields can be transformed to the form

$$Z_{YM}(\beta) = v_G^{-L} \int \prod_{\mu,n} dU_\mu(n) e^{-\beta S_W} Z_b(\beta) Z_f(\beta) = \tag{12}$$

$$= v_G^{-L} \int \mathcal{D}U_\mu \mathcal{D}\psi^* \mathcal{D}\psi \mathcal{D}\varphi \mathcal{D}\phi e^{-\beta(S_W + S_b + S_f)} \ , \tag{13}$$

where S_W is the Wilson action of gauge fields, v_G a volume of the group manifold G, L a number of lattice sites, and $\mathcal{D}$ denotes a product of corresponding field differentials over lattice sites. The effective action

$$S_{eff} = S_W + S_b + S_f \tag{14}$$

is invariant under gauge transformations (2)–(4) and (8), (9). The factor v_G^{-L} is included to cancel the gauge group volume factorizing upon the integration over field configurations in (13).

Now we may take the advantage of having scalar fields in the adjoint representation and restrict the gauge symmetry to the Cartan subgroup without imposing gauge conditions on the link variables. We make a change of the integration variables in (13)

$$\phi(n) = \tilde{g}(n) h(n) \tilde{g}(n)^{-1} \ , \tag{15}$$

where $\tilde{g}(n)$ belongs to the coset space G/G_H, $\dim G/G_H = \dim G - \dim G_H$, and $h(n) \in H$. Other new fields denoted $\tilde{U}_\mu(n)$, $\tilde{\varphi}$ and $\tilde{\psi}^*$, $\tilde{\psi}$ are defined as the corresponding gauge transformations of the initial fields with $g(n) = \tilde{g}^{-1}(n)$. No restriction on their values is imposed.

Relation (15) determines a one-to-one correspondence between old and new variables if and only if $\tilde{g}(n) \in G/G_H$ and $h(n) \in K^+$, where K^+ is the Weyl chamber in H. An element h of the Cartan subalgebra H belongs to the Weyl chamber $K^+ \subset H$ if for any simple root ω, $(h, \omega) > 0$; $(,)$ stands for an invariant scalar product in X. In a matrix representation of X, it is proportional to tr (see [16], pp. 187-190). With the help of the adjoint transformation, any element of a Lie algebra can be brought to the Cartan subalgebra. Since the Cartan subalgebra is invariant under the adjoint action of the Cartan subgroup, $\tilde{g}(n)$ must be restricted to the coset G/G_H. There are discrete transformations in G/G_H which form the Weyl group W [16]. Any element of W is a composition of reflections in hyperplanes orthogonal to simple roots in H. Its action maps H onto H itself. The Weyl group is a maximal isomorphism group of H [16]. Therefore, a one-to-one correspondence in (15) is achieved if $h(n) \in H/W \equiv K^+$.

Due to the gauge invariance of both the measure and exponential in (13), the integral over group variables $\tilde{g}(n)$ is factorized and yields a numerical vector that, being divided by v_G^L, results in $(2\pi)^{-Lr}$, $r = \dim H = \text{rank}\, G$. This factor is nothing but a volume of the Cartan gauge group G_H. The integration over $h(n)$ inquires a nontrivial measure, and the integration domain must be restricted to the Weyl chamber K^+. So, in (13) we have

$$v_G^{-1} \int d\phi(n) = (2\pi)^{-r} \int\limits_{K^+} dh(n)\mu(n) \ . \tag{16}$$

The measure has the form [17]

$$\mu(n) = \prod_{\alpha > 0} (h(n), \alpha)^2 \ , \tag{17}$$

where α ranges all positive roots of the Lie algebra X. The Cartan subalgebra is isomorphic to an r-dimensional Euclidean space. The invariant scalar product can be thought as an ordinary vector scalar product in it. Relative orientations and norms of the Lie algebra roots are determined by the Cartan matrix [16]. The integration measure for the other fields remains unchanged.

For example, $G = SU(2)$, then $r = 1$, $\mu = h^2(n)$ where $h(n)$ is a real number because $H_{SU(2)}$ is isomorphic to a real axis. The Weyl chamber is formed by positive $h(n)$. The $su(3)$ algebra has two simple roots $\omega_{1,2}$ $(r = 2)$. Their relative orientation is determined by the Cartan matrix, $(\omega_1, \omega_2) = -1/2$, $|\omega_{1,2}| = 1$. The Weyl chamber is a sector on a plane (being isomorphic to $H_{SU(3)}$) with the angle $\pi/3$. The algebra has three positive roots $\omega_{1,2}$ and $\omega_1 + \omega_2$. So, the measure (17) is a polynom of the sixth order. Its explicit form is given by (28).

The field $h(n)$ is invariant under Abelian gauge transformations

$$g_H(n)h(n)g_H^{-1}(n) = h(n), \quad g_H(n) \in G_H \ . \tag{18}$$

Therefore, after integrating out the coset variables $\tilde{g}(n)$ in accordance with (16), we represent the partition function of Yang-Mills theory as a partition function of the effective Abelian gauge theory

$$Z_{YM}(\beta) = (2\pi)^{-Lr} \int \mathcal{D}\tilde{U}_\mu e^{-\beta S_W} F(\tilde{U}) \ , \tag{19}$$

where

$$F(\tilde{U}) = (\det \beta D_\mu^T D_\mu)^{1/2} \int\limits_{K^+} \prod_n (dh(n)\mu(n))\, e^{-\beta S_H} \ , \tag{20}$$

$$S_H = 1/2 \sum_{n,\mu} tr \ \left(h(n+\hat{\mu}) - \tilde{U}_\mu^{-1}(n) h(n) \tilde{U}_\mu(n) \right)^2 \ . \tag{21}$$

To obtain (19), we have done the integral over both the Grassmann variables and the boson ghost field $\tilde{\varphi}(n)$, which yields $(\det \beta D_\mu^T D_\mu)^{1/2}$.

The function $F(\tilde{U})$ is invariant only with respect to Abelian gauge transformations, $\tilde{U}_\mu(n) \to g_H(n)\tilde{U}_\mu(n)g_H^{-1}(n+\hat{\mu})$. It provides a dynamical reduction of the full gauge group to its maximal Abelian subgroup. Since no explicit gauge condition is imposed on the link variables $\tilde{U}_\mu(n)$, the theory do not have usual gauge fixing deceases, like the Gribov copies or horizon. We shall call the Abelian projection thus constructed a *dynamical Abelian projection*.

3. Making a coset decomposition of the link variables [8]

$$\tilde{U}_\mu(n) = U_\mu^H(n) U_\mu^{ch}(n) \ , \tag{22}$$

where $U_\mu^H(n) = \exp u_\mu^H(n)$, $u_\mu^H(n) \in H$ and $U_\mu^{ch}(n) = \exp u_\mu^{ch}(n)$, $u_\mu^{ch}(n) \in X \ominus H$, we conclude that lattice Yang-Mills theory is equivalent to an Abelian gauge theory with the action

$$S_A = S_W - \beta^{-1} \ln F \ . \tag{23}$$

The link variables $U_\mu^{ch}(n)$ play the role of charged fields, while $U_\mu^H(n)$ represents "electromagnetic" fields. In the naive continuum limit, U_μ^H become Abelian potentials

$$U_\mu^H(n) \to \exp \int\limits_n^{n+\hat{\mu}} dx^\mu A_\mu^H \ , \qquad A_\mu^H \in H \ . \tag{24}$$

Note that the field $h(n)$ carries no Abelian charge and does not interact with U_μ^H as easily seen from (22) and (21) because $(U_\mu^H)^{-1}(n)h(n)U_\mu^H(n) = h(n)$.

Bearing in mind results on simulations of the Polyakov loop dynamics on the lattice, one should expect that the Coulomb field of charges in the effective Abelian theory is squeezed into stable tubes connecting opposite charges. A mechanism of the squeezing has to be found from a study of dynamics generated by (23). First, one should verify if the dual mechanism can occur in the effective Abelian theory.

In our approach, configurations $U_\mu^H(n)$ containing monopoles can exist. Kinematical arguments for this conjecture are rather simple. Let G be SU(N). In a matrix representation, the change of variables (15) becomes singular at lattice sites where the field $\phi(n)$ has two coinciding eigenvalues. This condition implies three independent conditions on components of $\phi(n)$ which can be thought as equations for the singular sites. At each moment of lattice time, these three equations determine a set of spatial lattice vertices (locations of monopoles). Therefore on a four-dimensional lattice, the singular sites form world-lines which are identified with world-lines of monopoles [2]. The new link variables

$$\tilde{U}_\mu(n) = \tilde{g}(n) U_\mu(n) \tilde{g}^{-1}(n+\hat{\mu}) \tag{25}$$

inquires monopole singularities via $\tilde{g}(n)$. Their density can be determined along the lines given in [8].

So, monopole dynamics is the dynamics of configurations $\phi(n)$ with two equal eigenvalues in the full theory (13). If such configurations are dynamically preferable, then one can expect that in the dynamical Abelian projection, effective monopoles and antimonopoles form a condensate.

All monopole-creating configurations of the scalar field $\phi(n)$ can easily be classified in a gauge invariant way. First of all we observe that the change of variables (15) is singular if its Jacobian vanishes

$$\prod_n \mu(n) = 0 \ . \tag{26}$$

We have to classify all configurations $\phi(n)$ which lead to $\mu(n) = 0$. The polynom (17) is invariant with respect to the Weyl group. According to a theorem of Chevalley [16], any polynom in H invariant with respect to W is a polynom of basis (elementary) invariant polynoms $tr\ h^l(n)$ with $l = l_1, l_2, ..., l_r$ being the orders of independent Casimir operators of G [16]. Therefore,

$$\begin{aligned} \mu(n) &= P(tr\ h^{l_1}(n), tr\ h^{l_2}(n), ..., tr\ h^{l_r}(n)) = \\ &= P(tr\ \phi^{l_1}(n), tr\ \phi^{l_2}(n), ..., tr\ \phi^{l_r}(n)) = 0 \ . \end{aligned} \tag{27}$$

Solutions of this algebraic equation determine all configurations $\phi(n)$ which will create monopoles in the dynamical Abelian projection (19). For $G = SU(3)$, we have $r = 2,\ l_1 = 2,\ l_2 = 3$ and [18]

$$\mu_{su(3)}(n) = \frac{1}{2}\left(tr\ \phi^2(n)\right)^3 - 3\left(tr\ \phi^3(n)\right)^2 = 0 \ . \tag{28}$$

Note also that $\mu_{su(3)} \sim (\phi_1 - \phi_2)^2(\phi_2 - \phi_3)^2(\phi_3 - \phi_1)^2$ where $\phi_{1,2,3}$ are eigenvalues of the hermitian 3×3 matrix $\phi \in su(3)$.

A dynamical question is: whether such configurations are dynamically preferable in the full theory (13). If they are not, the squeezing of the electrical field cannot be explained by the dual mechanism because a creation of monopole-like excitations would be dynamically unfavorable. Studies of relative mean-values of gauge-invariant local operators like $tr\ \phi^k(n)$ and of $\mu(n) = P$ in the full theory (13) could answer this question. Since (27) determines all configurations of $\phi(n)$ which could create topological monopole-like excitations in the Abelian theory, the above investigation of dynamics would also show if these effective excitations are indeed relevant to the squeezing the Abelian electrical field and, hence, to the QCD confinement. Clearly, the approach is gauge invariant.

4. Dynamics of monopoles is described by configurations of an auxiliary field $\phi(n)$ satisfying the gauge invariant condition (27). As the field $\phi(n)$ is coupled to gauge fields in a standard (gauged) way, it is natural to find configurations of the link variables in the full theory (13) which turns into monopole-carrying configurations of U_μ^H in the dynamical Abelian projection. These configurations must be relevant for the confinement, provided the dual mechanism does occur in the dynamical Abelian projection.

As follows from (21) and (22), the Abelian field $U^H_\mu(n)$ and the Cartan field $h(n)$ are decoupled because $[U^H_\mu(n), h(n)] = 0$. So, in the full theory, we define Abelian link variables by the relation

$$[U^\phi_\mu(n), \phi(n)] = 0 \ . \tag{29}$$

The coset decomposition assumes the form

$$U_\mu(n) = U^\phi_\mu(n) U^{ch}_\mu(n) \ . \tag{30}$$

One can regard it as a definition of charged fields $U^{ch}_\mu(n)$ for given $U_\mu(n)$ and $\phi(n)$.

Consider a vector potential corresponding to $U^\phi_\mu(n)$ as determined by (24). It has the form

$$A^\phi_\mu(n) = \sum_{\alpha=1}^{r} B^\alpha_\mu(n) e^\phi_\alpha(n) \ , \tag{31}$$

where $B^\alpha_\mu(n)$ are real numbers, and Lie algebra elements $e^\phi_\alpha(n)$ form a basis in the Cartan subalgebra constructed in the following way

$$e^\phi_\alpha = \lambda_i tr\ \lambda_i \phi^{l_\alpha - 1} \ . \tag{32}$$

It is not hard to be convinced that [18]

$$[e^\phi_\alpha,\ e^\phi_\beta] = 0 \ . \tag{33}$$

Since for any group G one of the numbers l_α is equal to 2, one of the elements (32) coincides with ϕ itself. The elements (32) are linearly independent in X because

$$\det P_{\alpha\beta} \equiv \det tr\ e^\phi_\alpha e^\phi_\beta = const \cdot P \ . \tag{34}$$

So, a generic element ϕ of X has a stationary group $G_\phi \subset G$ with respect to the adjoint action of G in X, $g_\phi \phi g_\phi^{-1} = \phi,\ g_\phi \in G_\phi$. This stationary group is isomorphic to the Cartan subgroup G_H. All linear combinations of the elements (32) form a Lie algebra of $G_\phi \sim G_H$.

In fact, the basis (32) can be constructed without an explicit matrix representation of λ_i. We recall that for each compact simple group G and its Lie algebra X, there exist $r = \mathrm{rank}\, G = \dim H$ symmetrical irreducible tensors of ranks l_α, $d_{i_1,i_2,\dots,i_{l_\alpha}}$, invariant with respect to the adjoint action of G in X. Clearly, $(e^\phi_\alpha)_i = d_{ij_1\dots j_{l_\alpha-1}} \phi_{j_1} \cdots \phi_{j_{l_\alpha-1}}$.

Now it is easy to see that the Abelian potentials $B^\phi_\mu(n)$ are singular at lattice sites where $\phi(n)$ satisfies (27). Indeed, from (31) we get

$$B^\alpha_\mu(n) = P^{\alpha\beta}(n) tr\ e^\phi_\beta(n) A^\phi_\mu(n) \ , \tag{35}$$

where $P^{\alpha\beta} P_{\beta\gamma} = \delta^\alpha_\gamma$. The determinant of the matrix $P_{\alpha\beta}(n)$ vanishes at the sites where $\mu(n) = P(n) = 0$. At these sites, the inverse matrix $P^{\alpha\beta}(n)$ does not exist, and the fields $B^\alpha_\mu(n)$ are singular. For unitary groups SU(N), $l_\alpha = 2, 3, ..., N$, the singular sites form lines in the four-dimensional lattice [2],[8]. These lines are world-lines of monopoles.

5. The above procedure of avoiding explicit gauge fixing can be implemented to remove the gauge arbitrariness completely and, therefore to study gauge-variant correlators,

like the gluon propagator, or some other quantities requiring gauge fixing on the lattice [20]. The advantage of dynamical gauge fixing is that it is free of all usual gauge fixing dynamical artifacts, Gribov's ambiguities and horizon [14]. It is also Lorentz covariant.

Recent numerical studies of the gluon propagator in the Coulomb gauge [19] show that it can be fit to a continuum formula proposed by Gribov [9]. The same predictions were also obtained in the study of Schwinger-Dyson equations where no effects of the Gribov horizon have been accounted for [13]. The numerical result does not exclude also a simple massive boson propagator for gluons [19]. So, the problem requires a further investigation.

Gauge fixing singularities (the Gribov horizon) occur when one parametrizes the topologically nontrivial gauge orbit space by Cartesian coordinates. So, these singularities are pure kinematical and depend on the parametrization (or gauge) choice. They may, however, have a dynamical evidence in a gauge-fixed theory [21]. For example, a mass scale determining a nonperturbative pole structure of the gluon propagator in the infrared region (gluon confinement) arises from the Gribov horizon [9], [12] if the Lorentz (or Coulomb) gauge is used. From the other hand, no physical quantity can depend on a gauge chosen. There is no gauge-invariant interpretation (or it has not been found yet) of the above mass scale. That is what makes the gluon confinement model based on the Gribov horizon looking unsatisfactory.

Here we suggest a complete dynamical reduction of the gauge symmetry in lattice QCD, which involves no gauge condition imposed on gauge fields and, hence, is free of the corresponding kinematical artifacts.

For the sake of simplicity, we discuss first the gauge group SU(2). Consider two auxiliary (ghost) complex fields ψ and ϕ, Grassmann and boson ones, respectively. Let they realize the fundamental representation of SU(2), i.e. they are isotopic spinors. The identity (11) assumes the form

$$Z_b(\beta)Z_f(\beta) = \int \mathcal{D}\phi^+\mathcal{D}\phi\mathcal{D}\psi^+\mathcal{D}\psi e^{-\beta(S_b+S_f)} = 1 \ , \tag{36}$$

where $S_f = \sum_n(\nabla_\mu\psi)^+\nabla_\mu\psi$ and $S_b = 1/2\sum_n(\nabla_\mu\phi)^+\nabla_\mu\phi$, and the lattice covariant derivative in the fundamental representation is defined by $\nabla_\mu\phi(n) = \phi(n+\hat{\mu}) - U_\mu^{-1}(n)\phi(n)$. Inserting the identity (36) into the integral representation of the Yang-Mills partition function (12), we obtain an effective gauge invariant action. The ghost fields are transformed as $\phi(n) \to g(n)\phi(n)$ and $\psi(n) \to g(n)\psi(n)$.

In the integral (13), we go over to new variables to integrate out the gauge group volume

$$\int d\phi^+(n)d\phi(n) = v_{su(2)}\int_0^\infty d\rho(n)\rho^3(n) \ , \tag{37}$$

where $\phi(n) = \tilde{g}(n)\chi\rho(n)$, $\chi^+ = (1\ 0)$, $\rho(n)$ is a real scalar field, and $\tilde{g}(n)$ is a generic element of SU(2). A new fermion ghost field and link variables $\tilde{U}_\mu$ are related to the old ones via a gauge transformation with $g(n) = \tilde{g}^{-1}(n)$. Since the effective action is gauge invariant, the integral over $\tilde{g}(n)$ yields the gauge group volume $v^L_{su(2)}$. We end up with the effective theory

$$Z_{YM}(\beta) \ = \ \int \mathcal{D}\tilde{U}_\mu e^{-\beta S_W}F(\tilde{U}) \ , \tag{38}$$

$$
\begin{aligned}
F(\tilde{U}) &= (\det \beta \nabla_\mu^+ \nabla_\mu)^{1/2} \int_0^\infty \prod_n \Big(d\rho(n)\rho^3(n) \Big)\, e^{-\beta S(\rho)} \ , && (39) \\
S(\rho) &= 1/2 \sum_{n,\mu} \Big(\rho(n+\hat{\mu}) - \chi^+ \tilde{U}_\mu^{-1}(n) \chi \rho(n) \Big)^2 \ . && (40)
\end{aligned}
$$

The function (39) is not gauge invariant and provides the dynamical reduction of the SU(2) gauge symmetry. A formal continuum theory corresponding to (38) has been proposed and discussed in [14].

Expectation values of a gauge-variant quantity $G(U)$ are determined by

$$
\langle G(U) \rangle \equiv \langle F(U) G(U) \rangle_W = \int \mathcal{D}U_\mu e^{-\beta S_W} F(U) G(U) \ . \tag{41}
$$

For example, for the gluon two-point correlator one sets $G(U) = A_\mu(n) A_{\mu'}(n')$ where the gluon vector potential on the lattice reads

$$
2iaA_\mu(n) = U_\mu(n) - U_\mu^+(n) - \frac{1}{2} tr \ (U_\mu(n) - U_\mu^+(n)) \tag{42}
$$

with a being the lattice spacing.

For gauge groups of higher ranks, like SU(N), the dynamical reduction can be done in a few steps as suggested in [14]. Note that the procedure (36)–(40) being applied to SU(N) would reduce the gauge symmetry to SU(N-1). To reduce the gauge symmetry completely, one should repeat the procedure $N-1$ times.

Another simpler way is to start with the dynamical Abelian projection (19). The Abelian G_H-symmetry can be reduced in a way similar to (36)–(40), or one can just impose the Lorentz gauge on the Abelian potentails (link variables). The latter procedure is exempt of Gribov's copying. Thus, in this approach, a dynamical reduction of any gauge symmetry group can be done in two steps.

Due to a complicated function (39) involved in (41), numerical simulations are harder to carry out than for a usual gauge fixing procedure. However, they could shed a light on the origin of the nonperturbative pole structure of the gluon propagator. In this approach, the gauge-dependent influence of the gauge fixing singularities, like Gribov horizon, on the gluon propagator poles is excluded.

A relation of (19) and (38) to the corresponding path integral representation of them with usual gauge fixing on the lattice can be established by making a change of variables. If one has to transform the integral (38) to the integral in a gauge $f(U) = 0$, the change of variables should have the form $\tilde{U}_\mu(n) = g(n) U_\mu^f(n) g^{-1}(n+\hat{\mu})$, $\phi(n) = g(n)\chi\rho(n)$ where $f(U^f) \equiv 0$. The measure reads $\int \mathcal{D}\tilde{U}_\mu \int_0^\infty \mathcal{D}\rho \prod_n \rho^3(n) = \int_{\Lambda_f} \mathcal{D}U_\mu^f \Delta_{FP}(U^f) \int \mathcal{D}\phi$ where Δ_{FP} is the Faddeev-Popov determinant in the gauge chosen and Λ_f is the fundamental modular domain for it. The integral over ϕ can be done and cancels the determinant in (39). So, the integral (38) assumes a usual gauge-fixed form. This establishes also a relation of the above approach with a standard gauge-fixed perturbation theory.

6. An extension of our approach to the full lattice QCD does not meet any difficulty because quarks would be decoupled with the ghost fields.

Though the integration domain is restricted in the sliced path integral (20), this restriction will disappear in the continuum limit because of contributions of trajectories reflected from the boundary ∂K^+ [17], [18]. It is rather typical for gauge theories that a scalar product for physical states involves an integration over a domain with boundaries which is embedded into an appropriate Euclidean space. The domain can even be compact as, for example, in two-dimensional QCD [22]. In the path integral formulation, this feature of the operator formalism is accounted for by appropriate boundary conditions for the transition amplitude (or the transfer matrix) rather than by restricting the integration domain in the corresponding path integral [22], [23]. In turn, the boundary conditions are to be found from the operator formulation of quantum gauge theory [18], [22], [23]. So, a study of the continuum limit requires an operator formulation of the dynamical reduction of a gauge symmetry, which has been done in [15].

The dynamical Abelian projection can be fulfilled in the continuum operator formalism. The whole discussion of monopole-like singular excitations given in sections 3 and 4 can be extended to the continuum theory. So, it determines Lorentz covariant dynamics of monopoles free of gauge fixing artifacts. To study monopole dynamics in the continuum Abelian gauge theory, one has to introduce monopole-carrying gauge fields [24].

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